CONSISTENT ANALYSIS METHOD FOR LONG TERM EFFECTS IN COMPOSITE BRIDGES

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INTRODUCTION

Composite structures are widely used in bridge engineering, mostly combining steel girders with a cast in place concrete slab. Long-term effects due to creep and shrinkage of the concrete together with the specific structural behaviour make it imperative that appropriate techniques be used in the design and analysis process for considering these effects in accordance with sufficiently accurate theories.
Some of the arising problems and their solution in the design and analysis process of composite bridges will be described in this paper. The basic theory needed for the proper numerical modelling of long-term effects combined with the specific structural behaviour of the bridges will be outlined. A consistent solution is proposed for both linear elastic theory and non-linear theory.
The presented solution has been implemented in a commercially available computer program. This implementation into the bridge design software is briefly described. The system takes into account all types of quasi-permanent loading and the time, when it is applied on the structure. The structural calculation includes the computation of the effects due to creep and shrinkage in the time intervals between activating new structural components and applying major new loadings. With such tools, the structural engineer is able to predict and follow the structural and material behaviour through all steps of bridge construction. A geometrical pre-processor and a powerful graphical post-processor facilitate the comparison of the behaviour and costs of different variants within short time.
Various composite bridges have already been successfully designed and analysed by applying the presented solution. The bridges presented in this paper as typical examples are suited to give insight into the problems specifically related to composite behaviour that were encountered and solved in the design process using the presented solution.

1 CREEP AND SHRINKAGE

In composite structures with concrete members, creep and shrinkage of concrete causes additional, time-dependent strain and stress within the steel members. The accurate design of steel composite structures requires numerical modelling of time dependent effects within design and analysis procedure.
The occurrence of time dependant plastic strain is a material property of concrete. It consists of two components, creep and shrinkage. Creep is a stress dependent material nonlinearity in which the material continues to deform under a constant load. Shrinkage does not depend on the load, and even an unloaded element will shrink.
The creep strain is generally expressed as a linear function of the elastic strain ε_e, i.e. $\varepsilon_{cr} = \varepsilon_e * \varphi$. Both, creep factor φ and shrinkage strain ε_s depend on a set of global properties denoted γ :

- global parameters (such as characteristics of the cement, quality of concrete, environmental data, average temperature, average humidity, material properties, etc.),
- cross-sectional section properties, average depth etc.,
- age of concrete

The creep factor depends on load application time, shrinkage is not dependent on load application time.

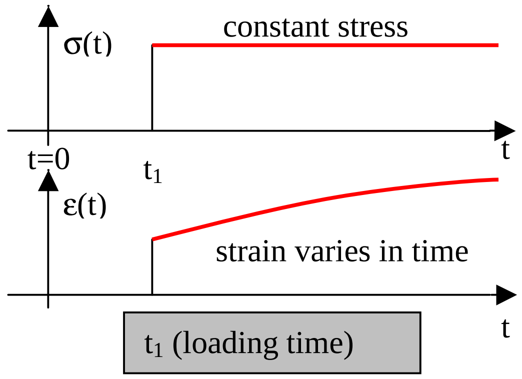


Fig. 1. Creep – development of the creep strain over the time under the constant stress

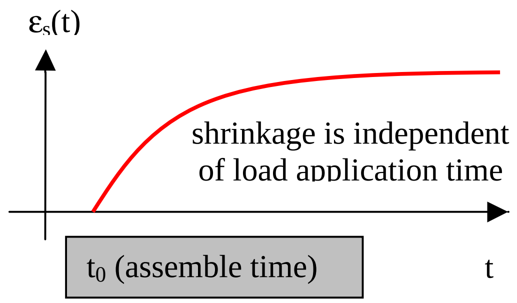


Fig. 2. Shrinkage – development of the shrinkage strain over the time

The total plastic strain is the sum of the creep strain and shrinkage strain as shown in *Eq. (1)*:

$$\varepsilon_{pl} = \varepsilon_e * \varphi + \varepsilon_s$$
$$\varepsilon_s = \varepsilon_s(t,\gamma) \qquad (1)$$
$$\varphi = \varphi(t,\tau,\gamma)$$

ε_{pl} … plastic strain due to creep and shrinkage
ε_e … elastic strain due to permanent load
ε_s … shrinkage strain depending on time t and global properties γ
φ … creep factor, depending on time t, load application time τ and global properties γ

The rules to determine the creep factor and shrinkage strain are very complex. Nowadays the CEB-FIP model is widely used. Many new design codes (EUROCODE, DIN, etc.) are based on these rules, with only minor differences to the original. Details are given in the CEB-FIP document [1].
The method used in this work is based on a time stepping scheme. The solution to the basic differential equation of the problem can now be written for the investigated creep interval [t_n, t_{n+1}] as follows:

$$\varepsilon_{t_{n+1}} = \varepsilon_{t_n} + \int_{\tau=0}^{t_n} \frac{1}{E}\cdot\frac{\partial\sigma_c}{\partial\tau}\cdot\left[\int_{t=t_n}^{t_{n+1}} \frac{\partial\varphi}{\partial t}(t,\tau,\gamma)\cdot dt\right]\cdot d\tau + \int_{t=t_n}^{t_{n+1}}\frac{\partial\varepsilon_s}{\partial t}(t,\gamma)\cdot dt + \int_{\tau=t_n}^{t_{n+1}}\frac{1}{E}\cdot\frac{\partial\sigma}{\partial\tau}\cdot\left[1+\int_{t=\tau}^{t_{n+1}}\frac{\partial\varphi}{\partial t}(t,\tau,\gamma)\cdot dt\right]\cdot d\tau \qquad (2)$$

$$\sigma_{t_{n+1}} = \sigma_{t_n} + \int_{t_n}^{t_{n+1}}\frac{\partial\sigma_c}{\partial t}\cdot dt$$

where: ε_{t_n} …… is known total strain at begin of investigated creep period [t_n, t_{n+1}];
σ_c …… concrete stress at actual time
φ …… known creep factor for load application time τ and global properties γ;
ε_s …… is known shrinkage function independent from load application time;
$\varepsilon_{t_{n+1}}$ …… is unknown strain at the end of investigated creep period [t_n, t_{n+1}];
σ_{t_n} …… is known total stress at begin of investigated creep period [t_n, t_{n+1}];
σ …… is unknown stress in investigated creep period [t_n, t_{n+1}];
$\sigma_{t_{n+1}}$ …… is unknown total stress at end of investigated creep period [t_n, t_{n+1}]

In *Eq. (2)* the first term of the right side represents the total strain at the begin of the time interval, the second term the additional creep strain in the time interval due to the stress increments prior to t_n, the third term the shrinkage strain in the time interval and the last term the creep strain induced by the stress change during the creep interval. The last term has well known creep recursion definition: change of stress depends on change of strain and vice versa.
Assuming that time step [t_n, t_{n+1}] is small enough we can rewrite *Eq. (2)* in algebraic form as follows in *Eq. (3)* and *Eq. (4)*:

$$\Delta\varepsilon_{t_n} = \varepsilon_{t_{n+1}} - \varepsilon_{t_n} \qquad \Delta\sigma_{t_n} = \sigma_{t_{n+1}} - \sigma_{t_n}$$

$$\Phi_i = \int_{t=t_n}^{t_{n+1}} \frac{\partial\varphi_l}{\partial t}(t,\tau_i,\gamma)\cdot dt \qquad \bar{\varepsilon}_s = \int_{t=t_n}^{t_{n+1}} \frac{\partial\varepsilon_s}{\partial t}(t,\gamma)\cdot dt \qquad \Phi_w = \int_{t=t_w}^{t_{n+1}} \frac{\partial\varphi}{\partial t}(t,t_w,\gamma)\cdot dt \tag{3}$$

$$\Delta\varepsilon_{t_n} = \sum_{l=1}^{n} \frac{\Delta\sigma_l}{E}\cdot\Phi_i + \bar{\varepsilon}_s + \frac{\Delta\sigma_{t_n}}{E}\cdot[1+\Phi_w] \tag{4}$$

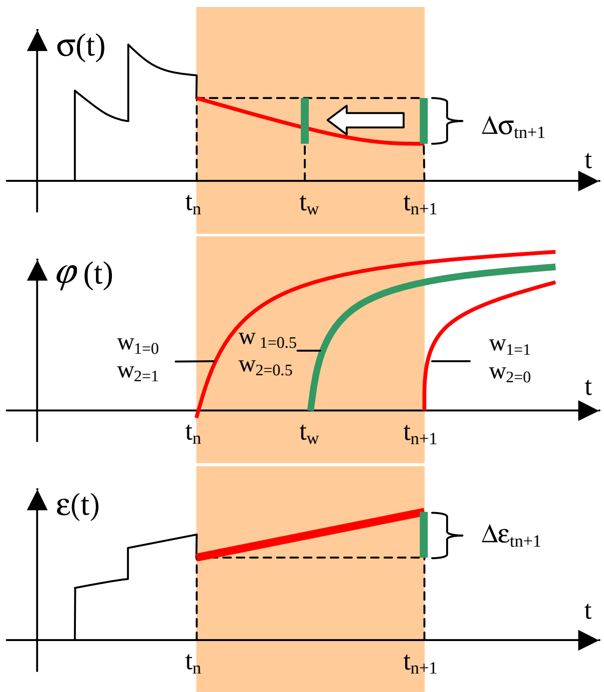
In *Eq. (4)* the problem is transformed into incremental form:

1. The first term is transformed from integral form into the finite sum of stress increments $l = 1,\dots, n$ at times τ_i prior to investigating creep period weighted with creep factors.

2. Second term is simply additional shrinkage strain.

3. The last term introduces weighting factors w_1 and w_2 into simple time step algorithm defining the influence of the elastic strain at beginning and end of the time step on the average value.

In *Eq. (5)* the meaning of weighting factors w_1 and w_2 is clarified.



$$t_w = t_n * w_2 + t_{n+1} * w_1$$
$$w_1 + w_2 = 1 \tag{5}$$

t_w is effective load application time for stress increment arising during the creep interval.

Total stress increment at the end of creep interval is put "back" in time to cover "creep of creep" effect. This novel approach allows for consistent storage of creep results as "normal loading case" result.

Elastic strain is defined as given in *Eq. (6)*:

$$\varepsilon^e(t) = \varepsilon^e_{t_n}\cdot\left(1-\frac{t-t_n}{\Delta t}\right) + (\varepsilon^e_{t_n} + \Delta\varepsilon^e_{t_{n+1}})\cdot\frac{t-t_n}{\Delta t}$$
$$\varepsilon^e(t) = \varepsilon^e_{t_n}\cdot w_2 + (\varepsilon^e_{t_n} + \Delta\varepsilon^e_{t_{n+1}})\cdot w_1 \tag{6}$$

Fig. 3 Stress, creep factor and strain development in the time interval t_n to t_{n+1}

3 EXPLICIT AND IMPLICIT CREEP ANALYSIS

The value of weighting factors w_1 and w_2 controls the quality of solution. For $w_1 = 0$, the solution degenerates to the explicit time integration (forward Euler method). The strain rate at begin of the time interval is assumed constant in the whole interval Δt. Very small time steps are required to minimize the error. Explicit approach is therefore not recommended for creep analysis.
The method becomes implicit for all weighting factors with $w_1 > 0$, and w1 = 1 refers to backward Euler integration method. Other proposals for implicit time integration schemes are in literature [2], e.g. the central difference scheme ($w_1 = w_2 = 0.5$) or the Galerkin scheme ($w_1 = 2/3$ and $w_2 = 1/3$). Schemes with w1 ≥ 0.5 are numerically unconditionally stable, i.e. they do not require very small time-steps. Implicit creep is generally more accurate, but accuracy is still dependent on the time-step length. A reasonable time-step must be used to capture the nonlinear creep behaviour accurately.

4 IMPLEMENTATION WITH ADJUSTED STIFFNESS

Eq. (4) can be solved for the unknown stress increment as follows:

$$\Delta\sigma_{t_n} = \frac{E}{1+\Phi_w} \cdot \left[\Delta\varepsilon_{t_n} - \sum_{l=1}^{n} \frac{\Delta\sigma_l}{E} \cdot \Phi_l - \bar{\varepsilon}_s \right] \tag{7}$$

Comparing *Eq. (7)* to engineering stress-strain definition we see that we can use exactly the same finite element formulation. The only difference is that for the creep loading case the elasticity modulus has to be replaced with E^{adj} [3] as shown in *Eq. (8) "Age adjusted effective modulus"*:

$$E^{adj} \equiv \frac{E}{1+\Phi} \tag{8}$$

The disadvantage of the implementation with adjusted stiffness is that creep effects cannot be mixed with other loadings and with non-linearity. Therefore the iterative method outlined in chapter 5 recommended and has been chosen for the implementation in the software package [4].

5 IMPLEMENTATION WITHIN NEWTON-RAPSHON

In the iterative Newton-Raphson method, creep due to stress increments arising during the time step is assumed as additional non-linearity. Iteration steps are applied until the convergence is achieved.

$$\left| \Delta\sigma_{t_n} - E \cdot \Delta\varepsilon_{t_n} + E \cdot \left[\sum_{l=1}^{n} \frac{\Delta\sigma_l}{E} \cdot \Phi_i + \bar{\varepsilon}_s + \frac{\Delta\sigma_{t_n}}{E} \cdot \Phi_w \right] \right| \leq Tolerance \tag{9}$$

In *Eq. (9)* unknown plastic strain due to the stress increments arising during the time step is not known. The straightforward solution is to estimate it by using the values from last iteration k-1:

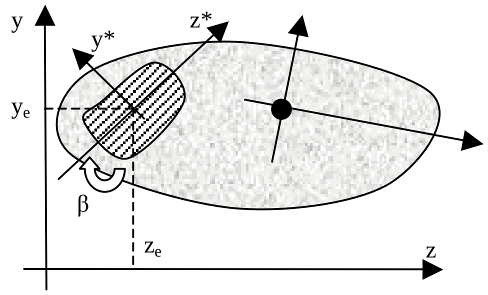
$$\left| \Delta\sigma_{t_n}^{k} - E \cdot \Delta\varepsilon_{t_n} + E \cdot \left[\sum_{i=1}^{n} \frac{\Delta\sigma_l}{E} \cdot \Phi_i + \bar{\varepsilon}_s + \frac{\Delta\sigma_{t_n}^{k-1}}{E} \cdot \Phi_w \right] \right| \leq Tolerance \tag{10}$$

The solution of *Eq. (10)* requires additional storage of previous iteration state. This can be avoided if we apply additional plastic strain in equilibrium calculation as shown in *Eq. (11)*.

$$\Delta\varepsilon_p = \frac{\Phi_w}{1+\Phi_w} \cdot \frac{\Delta\sigma_{t_n}^{k}}{E} \tag{11}$$

6 COMPOSITE EFFECTS - PRIMARY AND SECONDARY STRESSES

In composite structures – typically steel girders with concrete slabs – two types of material are combined within the deck section. Concrete layers with different ages act as different materials as they creep differently. The solution of the creep and shrinkage problem becomes more complicated because primary effects arise in addition as shown in *Fig 4* and *Fig 5*.



Creep material – in this case concrete – tends to change the volume during the creep period. Creep of concrete is constrained by the steel and therefore additional stresses will be introduced at cross section level even for globally static determinate systems. In first step the primary stresses are calculated at the cross section level [5].

Fig. 4. Each homogeneous part of the section has to be treated separately

The total stress in the composite section can be seen as the sum of primary and secondary stresses [6], [7], where the primary part represents the stresses due to the non-linear strain distribution in the cross-section plane, which are in equilibrium within the cross-section as shown in *Fig 5*.

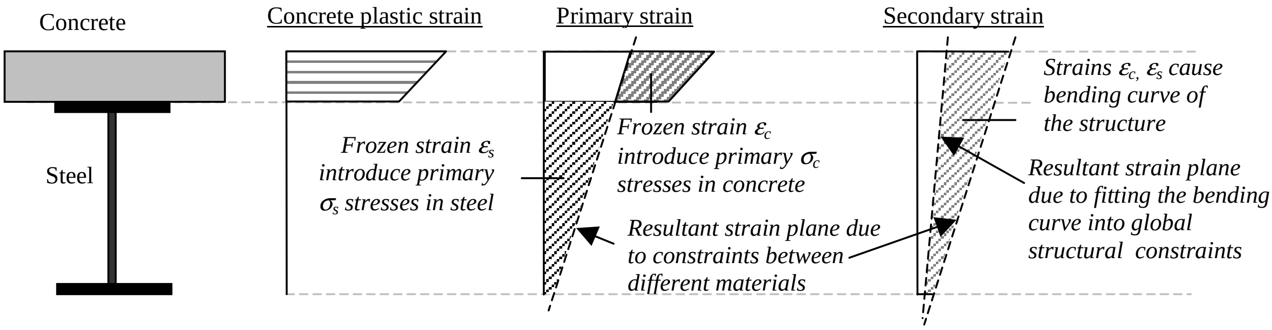


Fig. 5. Stress, creep factor and strain development in the time interval t_n to t_{n+1}

Applying the equilibrium condition on cross-section level leads to the actual strain plane resulting from constraints between different materials or different creep and shrinkage behaviour. Integrating these strains over the girder length results in a theoretical bending curve arising if the girder were statically determinate. Fitting the structure into external constraints (supports, constraints from other structural parts such as cross girders) yields additional strains and stresses in the composite girder.

$$\begin{aligned}
\sum N_x &\equiv \int_{(A_c)} \sigma \cdot dA_c + \int_{(A_s)} \sigma \cdot dA_s = 0 \\
\sum M_y &\equiv \int_{(A_c)} \sigma \cdot z \cdot dA_c + \int_{(A_s)} \sigma \cdot z \cdot dA_s = 0 \\
\sum M_z &\equiv \int_{(A_c)} \sigma \cdot y \cdot dA_c + \int_{(A_s)} \sigma \cdot y \cdot dA_s = 0
\end{aligned} \tag{12}$$

Eq. (12) shows the equilibrium conditions for the primary stresses. The three components refer to longitudinal strains and gradients in y and z direction. Creep due to shear is not considered (reasons see [8]). Stiffness properties in the local system are required for all homogeneous parts of the cross-section (A, Iy, Iz, β, e_y, e_z). *Eq. (13)* shows the principles of the primary state calculation:

$$\begin{aligned}
&\sum_{i=1}^{n} S_i^T \cdot T_i^T \cdot K_i^{adj} \cdot T_i \cdot S_i \cdot (\varepsilon - \varepsilon_i) = 0 \\
&F_i = K_i \cdot [\varepsilon - \varepsilon_i]
\end{aligned} \tag{13}$$

where:

$\varepsilon = \{\varepsilon_0 \quad \kappa_y \quad \kappa_z\}^T$ ……………….*resultant strain plane*

$\varepsilon_i = \sum_{l=1}^{n} \varepsilon_l \cdot \phi_l + \varepsilon_{si}$ …….. *initial plastic strain for part I*

$F_i = \{N_x \quad M_y \quad M_z\}_i^T$ ……... *primary forces in part i*

$K_i = E^i \begin{bmatrix} A & 0 & 0 \\ 0 & Iy & 0 \\ 0 & 0 & Iz \end{bmatrix}_i$ ……..*stiffness matrix for part i*

$K_i^{adj} = (1+\phi_w) \cdot K_i$ …………. *age adjusted stiffness matrix*

$T_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta_i & \sin\beta_i \\ 0 & -\sin\beta_i & \cos\beta_i \end{bmatrix}_i$ ……………*Rotation matrix*

$S_i = \begin{bmatrix} 1 & 0 & 0 \\ z_e & 1 & 0 \\ -y_e & 0 & 1 \end{bmatrix}_i$ ………………………*Translation matrix*

The resultant "strain plane" ε is then applied on the overall structure like a temperature load. For statically non-determinate structures, the response of the structure to this application of the "strain plane" results in the "Secondary effects" covered in global finite element analysis of the structure.

7 STRESS TIME HISTORY

As the creep directly depends on the load application time and the corresponding static system, it is necessary to store all stress increments during the life of the structure - from time zero up to time infinity. This data are basis for the time dependent solution. Difficulties arise with storing stresses directly produced by the time effects; they represent continuous change in time instead of stress increment.

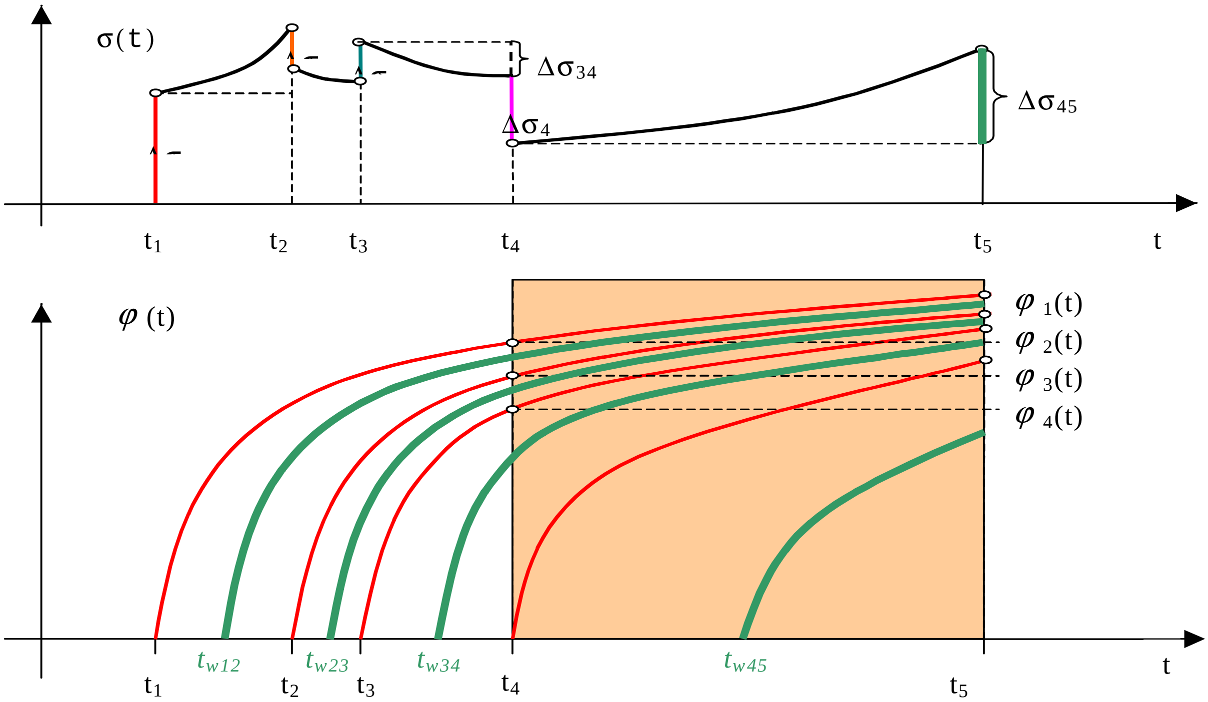


Fig. 6. Creep functions are different for different loading stress increments $\varphi_l(t)$, l=1,4

8 CONCLUSION

In this work a novel numerical implementation of creep problem in composite bridges is outlined. The presented solution allows for analysis of time dependent effects combined with other bridge loading and/or non-linear analysis problems, like second order effect, large displacement or cable sagging. The calculation of creep and shrinkage factors is fully automatic based on any modern design code. The engineer has to specify only general material properties and to define global environmental parameters. The solution converges to the exact solution, as time steps are refined.

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