Sophisticated Non-Linear 4D Structural Analysis for Cable Stayed Bridges

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INTRODUCTION

As the bridge engineering community sets sails to using longer and longer spans, more and more sophisticated analysis models have to be used in the design process. For large pre-stressed concrete bridges built using the incremental launching or free cantilevering methods, this applies mainly to accurately modeling the erection process in time, with considering the different construction stages, the time dependent behavior (creep and shrinkage), and the required pre-cambering to achieve the design shape after the construction process has been finished. However, the challenges for ultra long-span bridges such as stay cable or suspended bridges with high pylons and slender steel or concrete decks, are mainly related to optimizing the stressing sequence of the cables, to the geometrically non-linear behavior of the structure and to dynamic problems such as wind-induced vibrations. Referring to geometric non-linearity, we realize, that using the linear elastic theory is mostly sufficient for small and medium size bridges. The accuracy of the linear results is usually adequate, since non-linearity effects are small and can be accounted for by safety factors or even be ignored. Some non-linearity effects can – if required – also be locally taken into account in post-processing (design check) procedures. However, when analyzing long-span bridges, especially cable suspended bridges, non-linearity effects often reach magnitudes, which cannot be neglected in the structural analysis. In recent times, the structural systems have become more and more complex. The distribution of forces is often influenced by lots of constraints, and the load bearing behavior is characterized by interactions between the different structural parts. Using simple computation procedures has therefore become impossible and economic restraints widely require automated processes.

TIME EFFECTS

In practical engineering, time effects (creep, shrinkage and relaxation) must be considered in the analysis. The occurrence of time dependent plastic strain is a material property of concrete. Total plastic strain consists of creep plastic strain and shrinkage plastic strain. Shrinkage of concrete does not depend on the load. The shrinkage coefficient can be determined without difficulties using the design code regulations.

The effect of the loading history on the strains in concrete is much more pronounced compared to the other materials. For instance, considerable strain changes (creep recovery) can continue for a very long time after removal of all loads. National design codes take effects of the past loading history into account. Concrete creep models are generally defined by separated creep factors for each stress increment in the loading history. Rather complicated calculation of creep factors for each stress increment is impossible for hand calculation but presents no difficulty for computer implementation. (Janjic, Pircher: [Dubrovnik, 2004])

The numerical solution of time effects is accomplished step by stepping in the time domain.

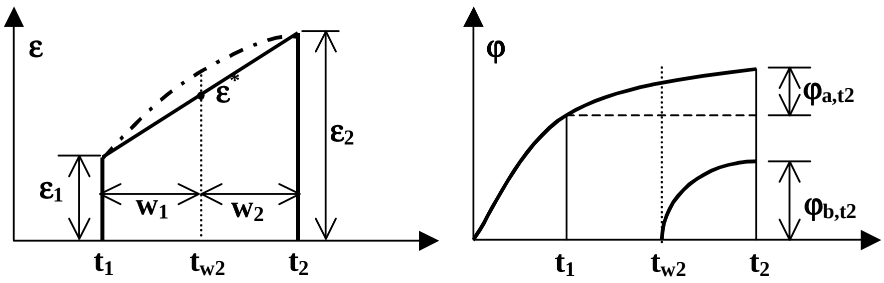

FIGURE 1 - ELASTIC STRAIN INCREMENT AND CREEP COEFFICIENT OVER TIME

Let time t1 corresponds to the start and time t2 to the end of a given time step. It can be assumed that complete load/response history is known (including all stress increments) from time equal to zero up to the start (time t1) of the time step. Basic unknowns are stresses and strains (or corresponding integral forces and displacement) at the end of time step (time t2). Instead of determining the continuous change between time t1 and time t2 it is advantageous to look only for the final solution at the end of the time step (at time t2). Linear change of elastic strain within time step according to the "finite differences" theory will be assumed.

The solution is to iterate the corresponding plastic strain due to the additional elastic strain increment within a general Newton/Raphson procedure. This solution is implemented in the software RM in a closed and consistent algorithm.

CONTINUOUS CHANGE OF STRUCTURAL SYSTEMS

Continuous change of structural systems is a major reason for getting non-linearity in structural analysis. To understand the physical reason, a very simple example of using a temporary support in the construction schedule is shown. Bending moments on the main girder before closure (with temporary primary support active) and after closure on the main girder due to temporary support removing are presented in Figure 2.

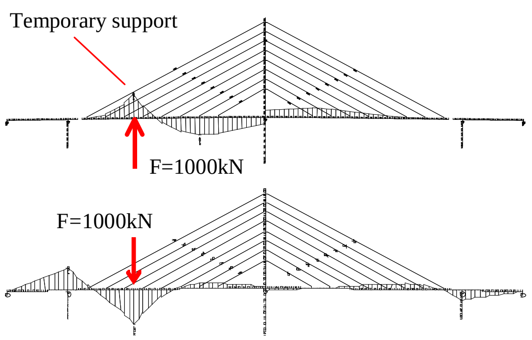

FIGURE 2 – BENDING MOMENTS BEFORE AND AFTER CHANGE OF STRUCTURAL SYSTEM

OPTIMIZATION

For cable stayed bridge design process special optimization procedures are necessary. The AddCon Method (The Additional Constraint Method. Bokan, Janjic, Heiden: [Budapest, 2006]) is a novel solution for optimization problems in structural engineering. The algorithm implemented in RM models every construction stage. The tensioning of each single cable is considered at first as a unit load case taking into account the current structural system and then influencing all previously applied unit load cases. All other loadings related to the individual erection procedure are also calculated step by step. All displacements and internal forces are accumulated and divided into one „constant“ and several „variable“ components. Each „variable“ component is related to one tensioning unit loading case and optimized in additional constraint module.

The results of one structural state i can be written as a vector of dimension e:

$$\left\{E^{0i}\right\} = \{E_1 E_2 E_3 \ldots E_e\}^T \tag{1}$$

Usually not only the basic results but also linear combinations of result vectors E_j are of interest. The significant results for the user can be written in vector form as well, where these results are calculated as linear combinations of the basic results:

$$\left\{E^{i}\right\} = [L]\ \left\{E^{0i}\right\} \tag{2}$$

$$\{E\} = \sum_{i=1}^{m} \left\{E^{i}\right\} \tag{3}$$

Vector $\{E^i\}$ has the dimension n where n << e. Matrix [L] has dimension n x e and converts result from vector $\{E^{0i}\}$ to vector $\{E^i\}$. The result value for which a constraint can be defined is calculated as the linear combination of all system state results. In each design step, a new, different result vector for the same chosen result will be produced. Parts of the results $\{E^i\}$ may be changed; other parts may be constant.

For the further analysis it is necessary to split the m result vector into mv variable and mc constant results (m = mc + mv). The mc constant results can be summed up directly. And all mv variable results can be written in matrix form:

$$\left\{E^{con}\right\} = \left\{E^{1,con}\right\} + \left\{E^{2,con}\right\} + \left\{E^{mc,con}\right\} \tag{4}$$

$$[M] = \left[\left\{E^{1,var}\right\}\left\{E^{2,var}\right\}\left\{E^{3,var}\right\}\ldots\left\{E^{mv,var}\right\}\right] \tag{5}$$

The sum of the constant results and the variable results should match the input constraints . The goal of the linear optimisation is to find which system parameter must be changed in order to meet the constraints. A vector with linear weight for the variable results describes the system parameters. Where the weighting factors {f} are the basic unknowns.

$$\left\{E^{input}\right\} = [M] * \{f\} + \left\{E^{con}\right\} \tag{6}$$

The constraint equation is a system of linear equations.

$$\{f\} = [M]^{-1} * \left(\left\{E^{user}\right\} - \left\{E^{const}\right\}\right) \tag{7}$$

In order to cover non-linear effects in the described system, the matrix [M] is split into a linear and a non-linear part. The non-linear part of Matrix [M] can be added to the constant results. The constant results get quasi-constant results (marked with an asterisk “*”):

$$\left\{{}^{*}E^{const}\right\} = [\Delta M] * \{f\} + \left\{E^{const}\right\} \tag{8}$$

$$\{E\} = [M_{LIN}] * \{f\} + \left\{{}^{*}E^{const}\right\} \tag{9}$$

In general case the matrix [M] depends on weighting factors {f}. This dependency is not given directly. One way of describing it mathematically is to produce changes in {f} and to watch corresponding changes in [E]. The unknown non-linear part of [M] can be computed by solving the linear system:

$$\begin{bmatrix} \frac{\partial M_{11}}{\partial f_1} & \frac{\partial M_{12}}{\partial f_2} & .. & \frac{\partial M_{1n}}{\partial f_n} \\ \frac{\partial M_{21}}{\partial f_1} & \frac{\partial M_{22}}{\partial f_2} & .. & \frac{\partial M_{2n}}{\partial f_n} \\ ... & ... & .. & ... \\ \frac{\partial M_{n1}}{\partial f_1} & \frac{\partial M_{n2}}{\partial f_2} & .. & \frac{\partial M_{nn}}{\partial f_n} \end{bmatrix} = \begin{bmatrix} \delta E_1^1 & \delta E_1^2 & ... & \delta E_1^n \\ \delta E_2^1 & \delta E_2^2 & ... & \delta E_2^n \\ ... & ... & ... & ... \\ \delta E_n^1 & \delta E_n^2 & ... & \delta E_n^n \end{bmatrix} * \begin{bmatrix} \delta f_1^1 & \delta f_1^2 & ... & \delta f_1^n \\ \delta f_2^1 & \delta f_2^2 & ... & \delta f_1^n \\ ... & ... & ... & ... \\ \delta f_n^1 & \delta f_n^2 & ... & \delta f_n^n \end{bmatrix}^{-1} \tag{10}$$

This equation is then the basis for a non-linear optimization solution. Each full calculation loop provides one additional piece of numerical information about the non-linear behavior. As the number of iterations grows, more and more data is available to find the proper non-linear part of [M]. The algorithm must find the best result to calculate from the solutions available (pivoting) and approximate all other elements of [M]. The more results that become available, the better the solutions.

PRE-CAMBER, ERECTION CONTROL

Depending on the erection sequence significant displacements occur during the construction of bridges. These displacements are compensated with pre-camber and specific fabrication shapes of girder components. The deformed structure is the start position for the calculation; therefore, full account of the current location in space is taken for the large deflection analysis. A precise structural analysis in construction stages is required to seriously control the erection geometry. The analysis is performed with a given starting geometry - usually the structure in Pre-camber is the geometry of the bridge required for assembly to reach the final geometry.

In the simple example in Figure 3 it is obvious that geometrical information is missing while assembling new element 3 and cable 10 to already assembled and loaded elements 1 and 2. The missing geometric information is automatically updated in the erection control mode as shown in Figure 3.

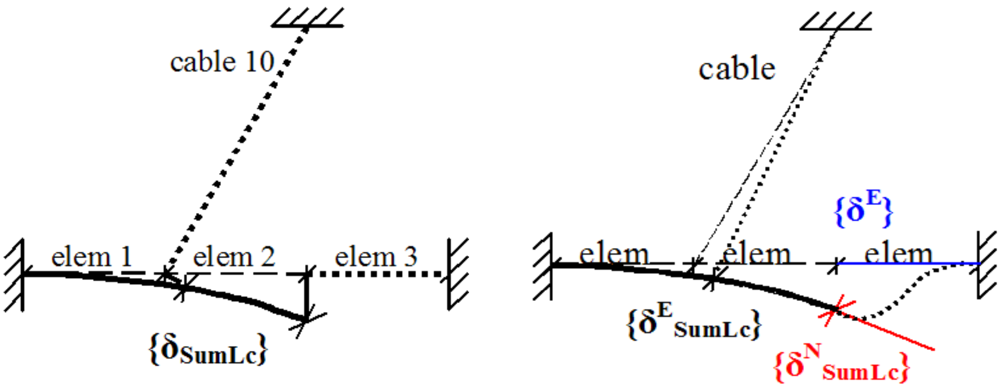

FIGURE 3 – GEOMETRICAL INFORMATION WHILE ASSEMBLING NEW ELEMENT 3 AND CABLE 10

Structural assembly in RM erection control mode (Janjic, Bokan: [Budapest, 2006])can be used to fully simulate certain construction conditions. With an option to automatically correct the kink at the segment face each newly-active element is fully constrained with a face-to-face connection to the currently active structure in its displaced position. As additional result, Erection control gives information if any force action is necessary to assemble the new segments. This allows determining any necessary equipment and possible construction problems already in the early design stage. It is important to note that both, linear and non-linear analyses are performed on the displaced structure in erection control mode, taking into account the exact geometrical lengths and rotations during the construction stages.

Since the stress-free fabrication shape is applied as a loading, acting on the currently active structure, the user can use this device to control and optimise the forces and displacements in the structure. The Fabrication Shape of cables is the ‘real’ stress free length of the cable as shown in Figure 4. The system length of a cable element is generally defined as the straight distance between the start and end-points before applying any transverse loading. Changing the stress-free length results in length differing from the system length of the element.

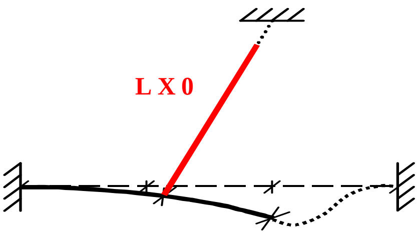

FIGURE 4 – STRESS FREE LENGTH OF THE CABLE IS CONSTRAINED IN CURRENT LENGTH BETWEEN ANCHORAGE AT BEGIN/END

For a beam segment, six input values are needed to define the stress free fabrication shape. Figure 5 shows the input as equivalent cantilever or simple beam.

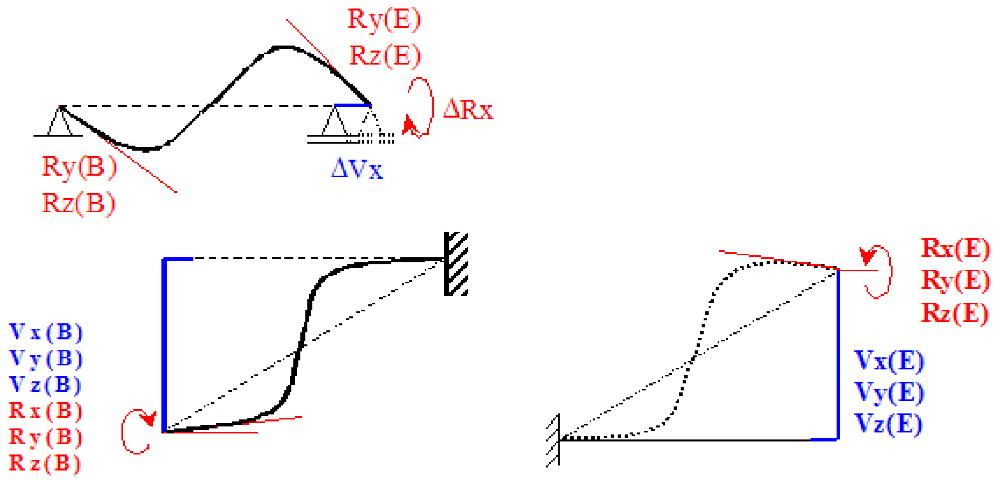

FIGURE 5 – FABRICATION SHAPE OF STRUCTURAL SEGMENTS INPUT AS EQUIVALENT CANTILEVER OR SIMPLE SPAN BEAM

The changing of the structural system is done automatically by the software by activating an option "Erection Control". The application of stress free fabrication shapes changes the stiffness matrix and adds additional loading terms.

NUMERICAL ANALYSIS

The mathematical model of a structure consists of “General Properties” describing physical parameters like the material behaviour, of the “Geometric Model”, describing the alignment of the structure in space as well as the cross-section geometry, and the “Time Model”, describing the construction sequence and the structural behaviour during life-time. Once the geometric model has been established, the model data are passed to the central analysis unit.

The time domain is considered from the very beginning by establishing a global time axis mirroring the time from the construction start during erection time and operation time to infinity. The different structural parts are activated and the loading is applied at the respective points in time corresponding to the construction process on site. The time dependent effects occurring in the intervals (creep and shrinkage) are fully considered on the correct structural system.

However, the “Schedule” is not only a reflection of the construction process on site, but a global framework defining the scheduled behaviour of the structure as well as the required investigations, optimisations and proof checks for any intermediate state during construction, and for the final stage after completion and at infinity. Variations of the schedule can be performed by modifying or adding “schedule actions” at the appropriate point in time.

APPLICATION EXAMPLE – SUTONG BRIDGE

The Suzhou-Nantong Yangtze River Bridge Project (Miao, Xiao, Pei, Zhang, Janjic, *et al.*: [New Delhi, 2004]) is located in the southeast of the Jiangsu Province in China. The total length of the bridge portion in this link is about 8.2 km. The design tasks of the project are carried out by China Highway Planning and Design Institute (HPDI) Consultants, Inc. in cooperation with Jiangsu Provincial Communication Planning & Design Institute, and the Architectural Design & Research Institute of Tongji University. The selected design for the Primary Fairway Bridge is a double-plane twin-pylon cable-stayed bridge with a continuous span arrangement of (100+100+300+1088+300+100+100) m, as shown in Figure 6. Two auxiliary piers and one transitional pier are erected in each side span. The main span of the bridge is 1088 m.

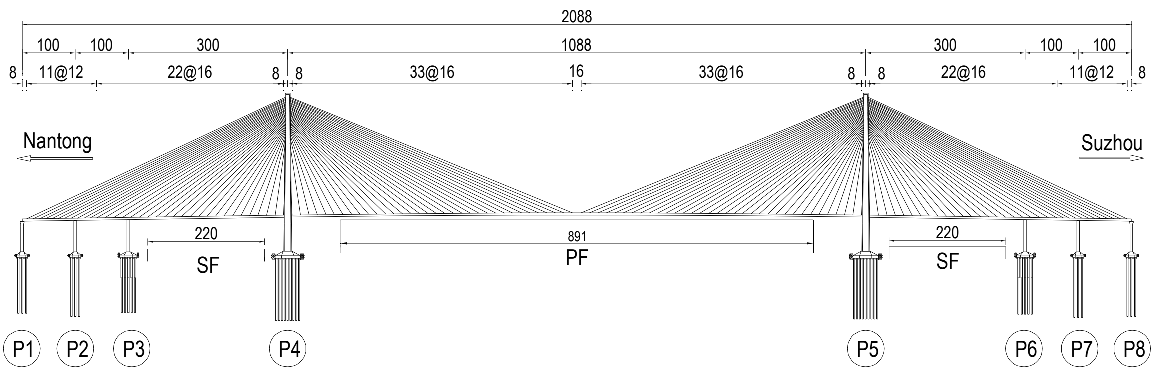

FIGURE 6 – SPAN ARRANGEMENT (UNIT: M)

The bridge girder is a streamlined closed flat steel box girder. The total width including wind fairing is 41.0 m accommodating dual 8 traffic lanes. The cross-section height is 4.0 m. The steel box is generally stiffened in the longitudinal direction with closed steel troughs. The standard cross-section of the girder is illustrated in Figure 7.

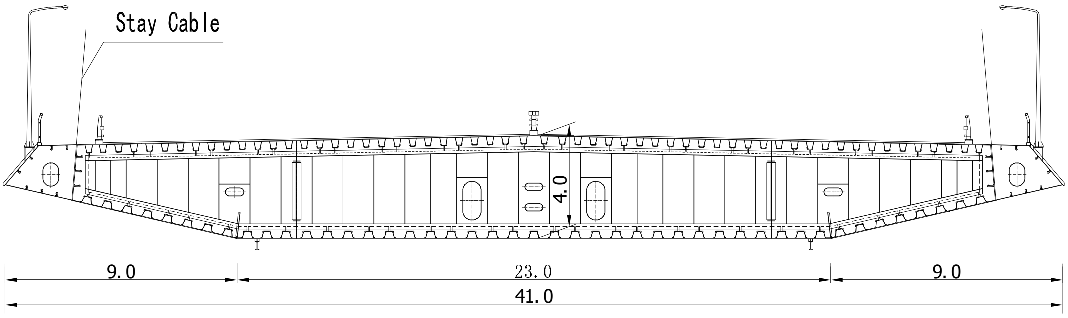

FIGURE 7 – CROSS SECTION OF THE GIRDER (UNIT: M)

The inverted concrete Y-shaped pylons are about 300 m in height. The stay-cables are anchored inside steel boxes fixed to the concrete by shear studs at the pylon top. The stay cables are arranged in double inclined cable planes with standard spacing of 16 m in the central span and 12 m near the ends of the back spans along the girder. The longest cable is about 577 m with a weight of 59 tons. The selected permanent connection between the girder and the pylons is accomplished by nonlinear dampers as used for the Great Belt East Bridge in Denmark. These dampers do not confine the displacement of the steel girder induced by temperature, moderate wind, and vehicle traffic, but transfer the loads from the girder induced by gust, earthquake and other forces from specific load combinations to an alternative pylon. A total of 4 dampers are placed at each pylon.

The RM program developed by TDV, Austria, has been used for the global analysis of the SuTong cable-stayed bridge in the detailed design. Two other programs named QJX and BAP have also been used for checking by the designer. The structural modelling keeps accordance with the planned construction schemes. Each of the stay-cables was divided into 8 sub-elements to consider cable-sag effects. Other interacting non-linear effects such as P-delta effect, large displacements and shear displacements were also considered in the calculation. Creep and shrinkage effects were calculated according to the CEB/FIP 90 code. The connection between the girder and both pylons were treated as nonlinear static spring elements with a gap value of 750 mm and a linear stiffness of 100 MN/m.

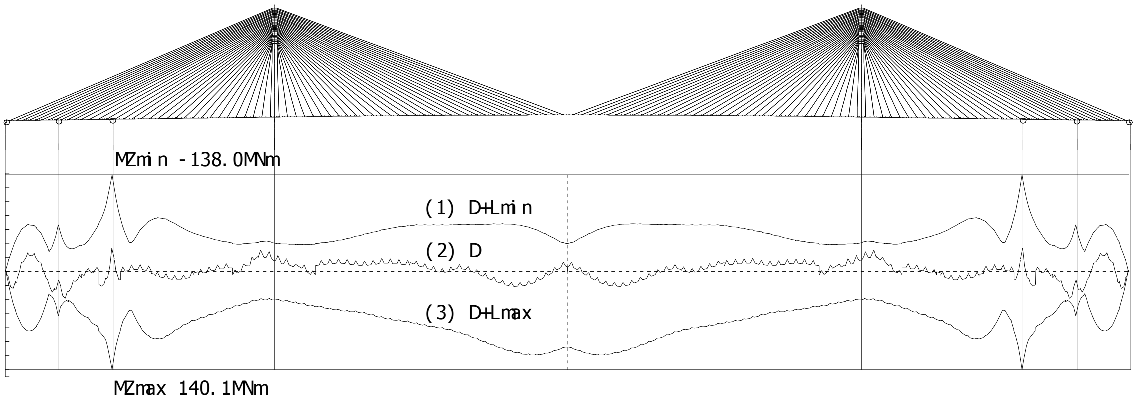

FIGURE 8 – BENDING MOMENT ENVELOP IN DECK

For the SuTong Bridge the contribution ratio of traffic loading is heavy for deck stress and counterweight arrangements in the back spans. The definition of the final state gives attention to both situations with and without traffic. The bending moment envelops in the deck under dead load and load combination 1 is shown in Figure 8.

The forward analysis with the ADDCON method implemented in RM was employed for all erection stages to achieve the final situation according to the construction schedules of the designer. All temporary supports, tie-downs, and movements of derricks for construction, temporary loading, and permanent loading were included in the model at various stages. The pre-camber calculation of all construction stages was computed by RM automatically. The third-order effect of pre-camber shapes apart from the design elevation of the deck was also considered approximately.

The results of the construction stage analysis also showed that the global stiffness of the bridge is very small before closure. For instance, the first tension of the longest stay cable in mid-span leads to a vertical deflection of 1.3 m at the end of the cantilever. Even after closure, the superimposed dead load (including paving, baluster, etc.) can still result in a vertical deflection of 1.8 m in the centre of mid-span. In addition, geometric nonlinearity in general has remarkable effects, especially for erection geometry of the deck.

The designer has given much attention to geometric nonlinearities all the way from the preliminary design phases to the detailed design. For the SuTong Bridge, compared with linear analysis, geometrically non-linear effects on the completed bridge may induce an offset of 10~20 percent of the maximum / minimum stress of the girder and the pylons under traffic loads, and the critical location can be also shifted.

CONCLUSION

A methodology to cover the needs of the design office and the contractor within one computer program has been implemented in RM program and presented. Using a finite difference in the time domain and Newton/Raphson for structural non-linearities the algorithm includes coupled effects of the creep, shrinkage and steel relaxation within overall structural non-linear analysis. Cable sagging, p-delta effects, large displacements and contact problems are combined with long term effects within consistent analysis.

The computer program automatically follows the bridge erection procedure and predicts the exact geometric position of structural segments taking into account the construction method, the construction sequence, the loading history, pre-stressing and post-tensioning, changes in support conditions and geometry and time-dependent effects.

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