Optimisation of the Tensioning Schedule for Cable-Stayed Bridges using Dynamic Software

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Summary

The design of long span cable-stayed bridges can prove tedious when it comes to finding an appropriate strategy for the stay cable tensioning procedure.
The design concept for achieving the appropriate tensioning procedure in cable stay bridges is often based on finding the forces in the individual cables that give rise to certain allowable structural displacements, moments or stress distributions in the girder and the pylons at the end of construction. The stressing forces and the sequence of stressing for all the cables needs to be optimised to meet these pre-defined requirements as closely as possible.
The calculation procedure described in this paper models every construction stage in detail with the tensioning of each individual cable being firstly considered as unit-loading cases acting on the currently active structural system and influencing all previously applied unit-loading cases.
The effects of the other loading cases appropriate to the construction procedure affecting all previously constructed parts (such as self weight of the new segment, traveller relocation etc.) are also calculated. The displacements and the internal forces from each construction stage are accumulated and the values are sub-divided into “constant” (i.e. self weight.) and several “variable” components - each “variable” component being connected to one of the unit loading cases.
A system of equations is built up by comparing these accumulated values with the initial design requirements. The result from the equation reduction is the intensity factors for all the unit-loading cases to achieve the predefined constraints (displacements, moments, stresses).
The dynamic software procedure works equally well for both linear and non-linear structures with the effects of Creep & Shrinkage being fully considered.
The benefit from this method is achieving an optimal tensioning strategy which results in reduced stressing actions with consequent huge time saving and cost saving during construction.
The concept is illustrated by the analysis of the Verige Bridge that crosses the Bay of Boka Kotorska in Montenegro.

Keywords: Cable Stayed Bridge, tensioning strategy, construction sequence, non-linear, software

1. Introduction

The AddCon Method (The Additional Constraint Method) is a novel solution for optimisation problems in structural engineering. This is an extension of the Unit Load Method for non-linear problems.
Bridge design and analysis is an iterative process. During this process the engineer is looking for the best solution for given criteria by changing specific system parameters. Engineer experience helps to reduce the time required, but there will still be a need for many iteration steps until the de-

sign criteria are met. Computer programs nowadays should provide the best possible support of this design process.
It should be mentioned here that it is not possible to have a computer program complete all engineering tasks. The best computer support that can be expected (even with a supercomputer) is to find the best solution for some given constraints, but it is still the engineer's duty to find correct and logical constraints and to prepare the input properly for the computer.
There are various constraints used in structural engineering. Most obviously, constraints can be applied to "calculation results" (deformations, stresses, forces, etc.). But also all other design parameters can be used as constraints, e.g. geometric parameters, material properties, etc. It is important that a computer supported design methodology supports various types of constraints.

2. Linear Optimisation of the Cable Tensioning

2.1 General

Standard bridge design processes begins with preparing a structural model, defining the loads and the construction schedule (construction stages). The engineer will then "run" a computer analysis. Two result types of the computer analysis are of main interest: The loading case result, and the envelope result. The loading case result represents the structural state at a definite time. The envelope result provides information about maximal/minimal peaks of a given result together with other corresponding results. Based on the results, the design criteria can be checked and optimisation can be started. A simple example is given to demonstrate the principle of optimised cable tensioning: In Fig.1 the bending moment diagram is shown for the final stage without optimisation of the cable tensioning. In Fig.2 the same results are shown after optimisation.

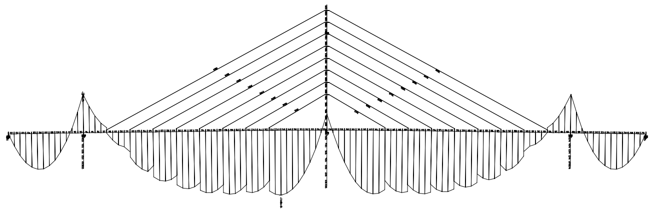


Fig.1 Bend. Moment Mz at final stage without optimisation

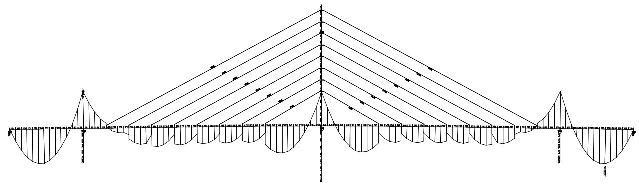


Fig.2 Bend. Moment Mz at final stage after optimisation

2.2 Result storage

The results of one structural state i (e.g result of loading case i) can be written as a vector of dimension e:

$$\left\{E^{0i}\right\} = \{E_1 E_2 E_3 .... E_e\}^T \quad (1)$$

Each item $E_j, j = 1..e$ in vector $\{E\}$ represents one result of any type, e.g. displacement, integral force/moment, stress, etc.
Usually not only the basic results but also linear combinations of result vectors E_j are of interest. The significant results for the user can be written in vector form as well, where these results are calculated as linear combinations of the basic results:

$$\left\{E^{i}\right\} = [L]\left\{E^{0i}\right\} \quad (2)$$

Vector $\{E^i\}$ has the dimension n where $n << e$. Matrix $[L]$ has dimension n x e and converts result from vector $\{E^{0i}\}$ to vector $\{E^i\}$. The result value for which a constraint can be defined is calculated as the linear combination of all system state results, e.g. as linear sum.

$$\{E\} = \sum_{i=1}^{m} \left\{E^{i}\right\} \quad (3)$$

Constant part: Mz due to permanent loads

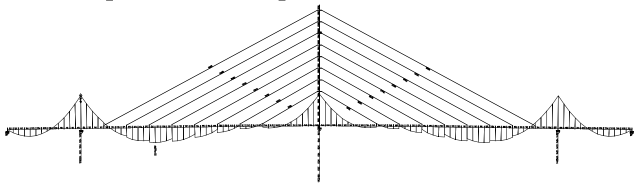


Fig.3 Single results Mz for additional dead loads at final stage (Linear calculation)

Variable part: Cable Stressing

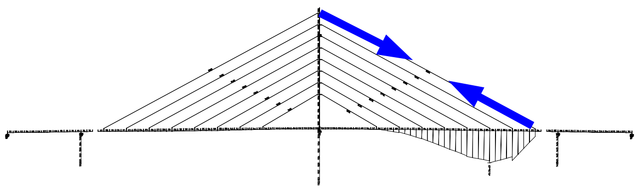


Fig.4 Forces, displacements, stresses etc. can be controlled with cable stressing

Stressing of Cables can be controlled

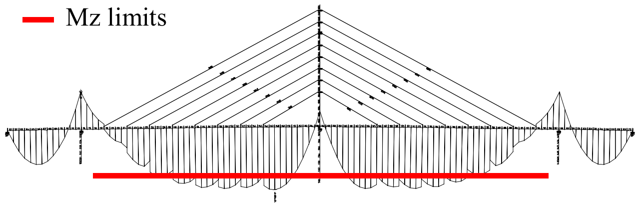


Fig.5 Bending moment Mz at final stage without optimisation

In each design step, a new, different result vector for the same chosen result will be produced. Parts of the results $\{E^i\}$ may be changed; other parts may be constant, depending on the different system parameters. For the further analysis it is necessary to split the m result vector into *mv* variable and *mc* constant results ($m = mc + mv$).

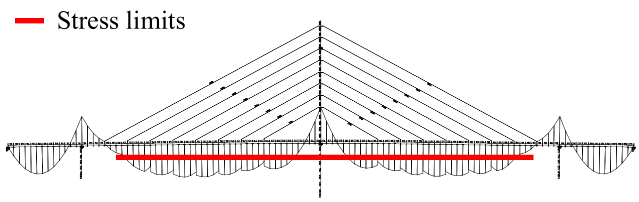


Fig.6 Stress results at final stage at the bottom edge of the main girder cross-section without optimisation

The *mc* constant results can be summed up directly:

$$\left\{E^{con}\right\} = \left\{E^{1,con}\right\} + \left\{E^{2,con}\right\} + \left\{E^{mc,con}\right\} \quad (4)$$

All *mv* variable results can be written in matrix form. Matrix [M] is of dimension n x mv:

$$[M] = \left[\left\{E^{1,\mathrm{var}}\right\}\left\{E^{2,\mathrm{var}}\right\}\left\{E^{3,\mathrm{var}}\right\}..\left\{E^{mv,\mathrm{var}}\right\}\right] \quad (5)$$

The sum of the constant results and the variable results should match the input constraints E^{input}. The goal of the linear optimisation is to find which system parameter must be changed in order to meet the constraints.

In the linear system model, each system parameter results in a variable result. A vector with linear weight for the variable results describes the system parameters. The constraint can now be written as shown in the following equation:

$$\left\{E^{input}\right\} = [M] * \{f\} + \left\{E^{con}\right\} \tag{6}$$

Where the weighting factors {f} are basic unknowns.

$$\begin{Bmatrix} M_z^{155} \rightarrow -20000 \\ M_z^{145} \rightarrow 4000 \\ M_z^{141} \rightarrow 4000 \\ M_z^{137} \rightarrow 4000 \\ \ldots \\ .. \\ .. \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c|c} \multicolumn{1}{c}{E_{1v}} & \multicolumn{1}{c}{E_{2v}} & \multicolumn{1}{c}{E_{3v}} & \multicolumn{1}{c}{E_{4v}} & \multicolumn{3}{c}{\ldots\ldots\ldots\ldots} \\ \hline 11823 & 4498 & 340 & -1572 & \ldots & \ldots & \ldots \\ \hline 0 & 7570 & 7659 & 5810 & \ldots & & \\ \hline 0 & 0 & 8065 & 8270 & \ldots & & \\ \hline 0 & 0 & 0 & 8227 & \ldots & & \\ \hline \ldots & \ldots & \ldots & \ldots & \ldots & & \\ \hline \ldots & & & & & & \\ \hline \ldots & & & & & & \end{array}\right] \times \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ .. \\ .. \\ .. \end{Bmatrix} + \begin{Bmatrix} -6325 \\ 5267 \\ 5638 \\ 5139 \\ \ldots \\ .. \\ .. \end{Bmatrix}$$

2.3 Solution of the constraints problem

The constraint equation is a system of linear equations. The solution is trivial for the special case where n=mv:

$$\{f\} = [M]^{-1} * \left(\left\{E^{user}\right\} - \left\{E^{const}\right\}\right) \tag{7}$$

It is necessary that matrix [M] is non-singular. If matrix [M] is singular, no physical solution exists. This may happen if the chosen constraints are coupled. Due to numerical effects, it is also possible to get solutions that have no practical meaning because matrix [*M*] is nearly singular.

3. Non-linear effects:

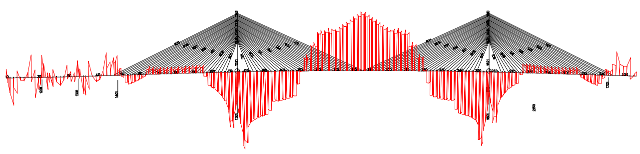
Provided that the results are calculated in a linear analysis, a linear optimisation solution can be expected, and for non-linear analysis a non-linear optimisation solution must be applied.

Unfortunately, this is not a case. Even with simple linear structural analysis, non-linear optimisation methods must be applied. There are two major reasons for the non-linear components in the results: Considering time effects in calculation and continuous changing of the structural system.

3.1 Considering time effects

In practical engineering, time effects (creep, shrinkage and relaxation) must be considered in the analysis. In most codes, the time effects are described to have “quasi superposition behaviour”. But these effects produce non-linear results in the final state. The reason for this is the forth dimension (time): A change of weighting factor for one loading will creep over time. These “creep results” will be increased with the same “weighting factor” if creep has “quasi superposition behaviour”. Looking at the structural state after some time, changes occur due to the initial change of loading and changes from all following creep steps up to this state. Changes due to creep are not known initially.
These non-linear effects change the results only slightly, but linear optimisation can not be applied any more. In order to cover these effects in the described system, the matrix [M] is split into a linear and a non-linear (time-effect) part:

$$[M] = [M_{LIN} + \Delta M] \tag{8}$$



Equation 6 changes to:

$$\{E\}=[M_{LIN}+\Delta M]*\{f\}+\{E^{const}\} \tag{9}$$

The non-linear part of Matrix [M] can be added to the constant results. The constant results get quai-constant results (marked with an asterisk "*"):

$$\{{}^{*}E^{const}\}=[\Delta M]*\{f\}+\{E^{const}\} \tag{10}$$

and Equation 9 changes to:

$$\{E\}=[M_{LIN}]*\{f\}+\{{}^{*}E^{const}\} \tag{11}$$

The difference of the constant parts of the results (which actually should be zero) is now a measure for the non-linear part of [M].

$$\{E^{err}\}=\{{}^{*}E^{const}\}-\{E^{const}\} \tag{12}$$

Fig.7 Optimised Analysis without creep and shrinkage effects for t=0

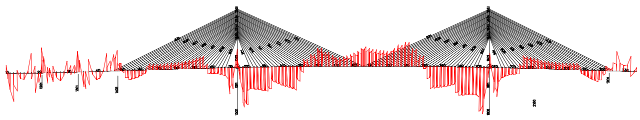


Fig.8 Results incl. Creep & shrinkage effects after an optimised linear calculation (t=∞, 85 years)

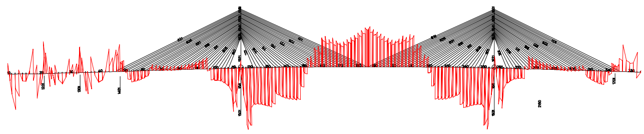


Fig.9 Optimised analysis including creep and shrinkage effects for the optimisation

3.2 Considering continuous change of structural systems

Continuous change of structural systems is another major reason for getting non-linear optimisation problems. To understand the physical reason, a very simple example of using a temporary cable in the construction schedule is shown in Fig. 10 & 11.
This non-linearity has a different nature to the time-effects non-linearity but it can be treated with the same optimisation method.

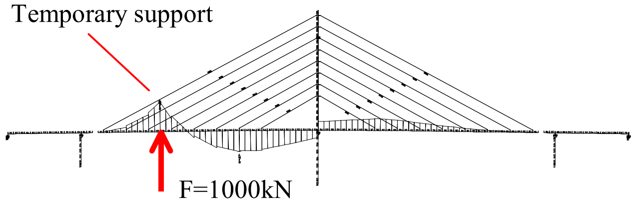


Fig.10 Bending moments on the main girder before closure (with temporary primary support active)

3.3 Non-linear structural behaviour

If structural response is not linear, the optimisation problem is non-linear from the very beginning. In practical cases this non-linearity is not too far away from a linear solution. Design experience shows that non-linear effects are usually within 20% of linear solution. This is the same order of magnitude as the non-linearity due to time effects. Again, these effects can be treated with the same method described above. With a mild non-linearity grade we cover almost all problems.

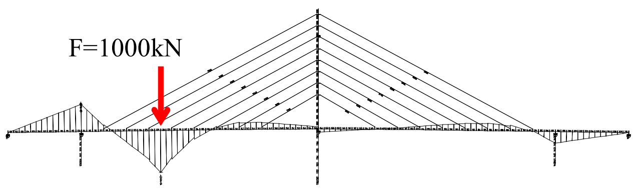


Fig.11 Bending moments after closure on the main girder due to temporary support removing

4. Non-Linear optimisation for "mild" non-linear problems

4.1 Linearization of the non-linear part

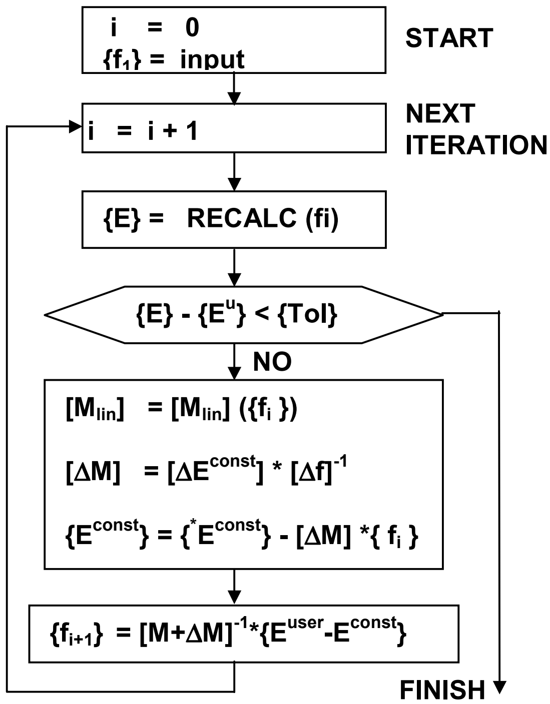
In general case the matrix $[M]$ depends on weighting factors $\{f\}$. This dependency is not given directly. One way of describing it mathematically is to produce changes in $\{f\}$ and to watch corresponding changes in $[E]$. The unknown non-linear part of $[M]$ can be computed by solving the linear system:

$$
\begin{bmatrix}
\frac{\partial M_{11}}{\partial f_1} & \frac{\partial M_{12}}{\partial f_2} & .. & \frac{\partial M_{1n}}{\partial f_n} \\
\frac{\partial M_{21}}{\partial f_1} & \frac{\partial M_{22}}{\partial f_2} & .. & \frac{\partial M_{2n}}{\partial f_n} \\
... & ... & .. & ... \\
\frac{\partial M_{n1}}{\partial f_1} & \frac{\partial M_{n2}}{\partial f_2} & .. & \frac{\partial M_{nn}}{\partial f_n}
\end{bmatrix}
=
\begin{bmatrix}
\delta E_1^1 & \delta E_1^2 & ... & \delta E_1^n \\
\delta E_2^1 & \delta E_2^2 & ... & \delta E_2^n \\
... & ... & ... & ... \\
\delta E_n^1 & \delta E_n^2 & ... & \delta E_n^n
\end{bmatrix}
*
\begin{bmatrix}
\delta f_1^1 & \delta f_1^2 & ... & \delta f_1^n \\
\delta f_2^1 & \delta f_2^2 & ... & \delta f_1^n \\
... & ... & ... & ... \\
\delta f_n^1 & \delta f_n^2 & ... & \delta f_n^n
\end{bmatrix}^{-1}
\tag{13}
$$

This equation is then the basis for a non-linear optimisation solution.

4.2 Iterative search for solution

Each full calculation loop provides one additional piece of numerical information about the non-linear behaviour. As the number of iterations grows, more and more data is available to find the proper non-linear part of [*M*]. The algorithm must find the best result to calculate from the solutions available (pivoting) and approximate all other elements of [*M*]. The more results that become available, the better the solutions. With a good selection algorithm, it is possible to find a solution for the non-linear problem for *n* unknown parameters within a fraction of *n* iteration steps.



During iteration step *i, k = i-1* differences for results and differences for factors are already available. The pivoting algorithm chooses the *k* – biggest lines from $\{E^{error}\}$ and extracts a *k* x *k* matrix [*dE**]. All other errors are approximated by a diagonal matrix [*dE***] consisting of the diagonal element of [*dE*]. The Matrix [*dM*] is then reconstructed out of the two results.

4.3 Convergence criteria

The iteration algorithm in the AddCon Method stops when the results are within a given tolerance $\{T\}$ of the design criteria:

$$\left|\{E^{user}\} - \{E\}\right| <= \{Tol\}$$

Changing of factors f over the iteration process can be seen in Fig.12.

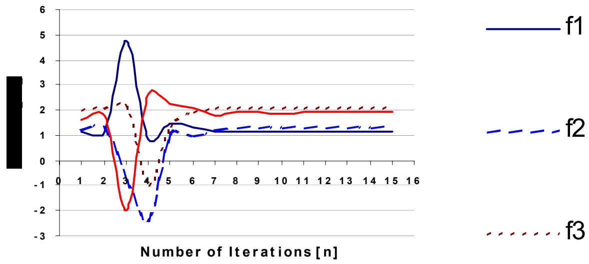


Fig.12 changing of the factors f over the iteration process (till user defined constraints are fulfilled)

5. Practical Application of the Unit Load Method: The Verige Bridge

5.1 General

All the project engineering work was performed by the Gradis, Slowenia. This company has used the TDV-software [1] including features described in [2,3] for many years in their construction offices for the design of different type of bridges. For the current project TDV and GRADIS had a close cooperation for all the electronic analysis work. Static and dynamic stability verification had been carried out by means of the software system RM2004 for all construction stages as well as for the final stage. With this program system a stage by stage analysis can be performed. In each calculation stage the results of all stages should be summed up. Thus a control of all internal forces and moments arising in a certain rime period is given. For concrete cross-sections pre-stressing with actual tendon characteristics and any geometry layout can be taken into account. For the design calculation of the pylon the 2nd order theory had been adopted with the software system RM2004. The construction stage analysis had been performed for overall 81 construction stages.

5.2 Project Description

The Verige Bridge is a cable-stayed bridge located in the southwest of Montenegro and crosses the Bay of Boka Kotorska. The bridge consists of 7 spans including the main cable-stayed part and the adjacent spans with a total length of 981m. The main span between the two pylons is 450m. The height of the bridge above sea level is about 50m.

Fig.13 Verige Bridge, spans of 41+50+90+130+450+130+90m, Montenegro [4]

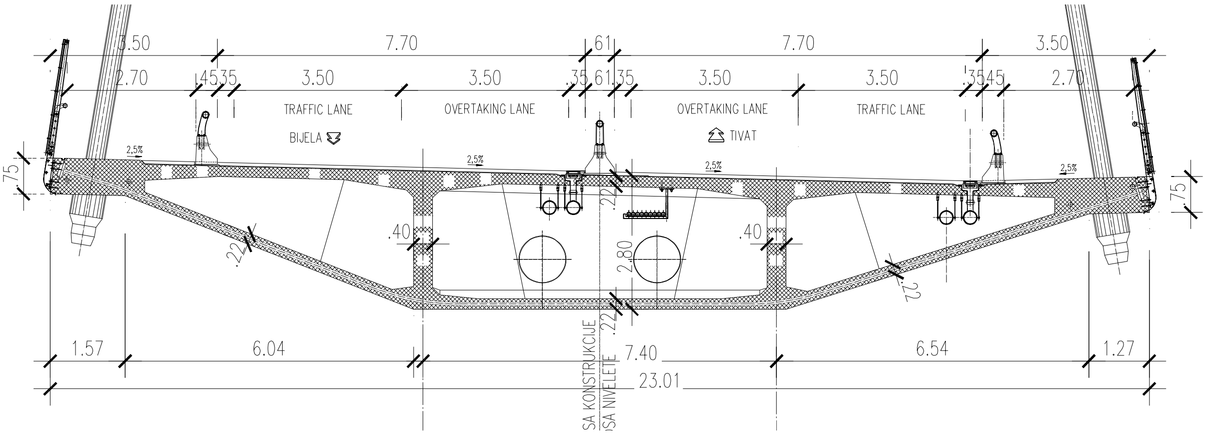


Fig.14 Verige Bridge, cross section [4]

5.3 Construction Stage Analysis

The cable-stayed bridge was to be erected by the cantilever method starting from both pylons in both directions until the approach-bridges were reached and a monolithic connection could be established after the closure of the main. The structural system for the final stage analysis as well as the structural system for the individual construction stages was modelled with RM2004 [1]. The main girder of the Cable-Stayed-Bridge was constructed symmetrically by free cantilevering. First, base parts above the piers of the approach spans were executed followed by a symmetrical construction of individual segments. At the same time, the first elements (hammer head) were constructed at both pylons as well from where free cantilevering was carried out. The length of an individual segment amounted to 5.0 metres. Every second segment was connected with a stay cable.

The remaining portion of the deck was also constructed according to the cast-in-situ free cantilever method while the deck portions at the abutment the piers of the approach spans were executed by means of a formwork.

Fig.15 to Fig.17 show the bending moment diagrams for different construction stages, including all the non-linear time-dependent effects, in the last iteration cycle of the cable stressing optimization procedure.

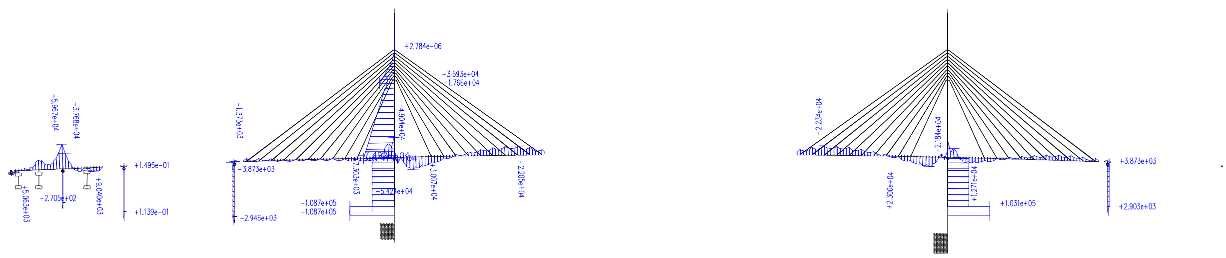


Fig.15 Construction stage 42 after stressing, bending moments in the superstructure

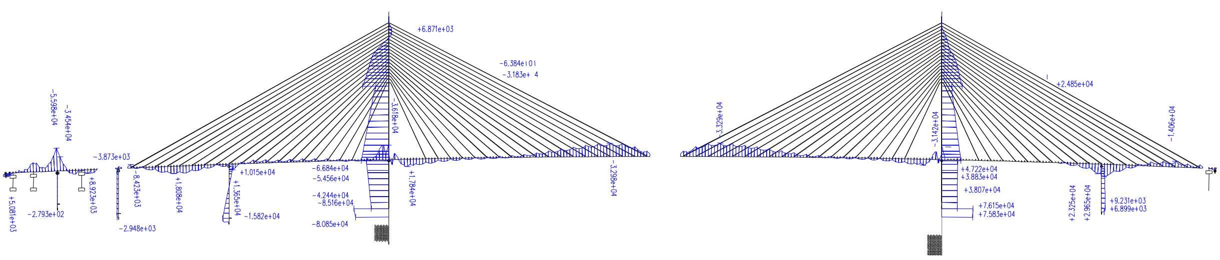


Fig.16 Construction stage 71 after closing of the approach spans 2 and 3, bending moments in the superstructure

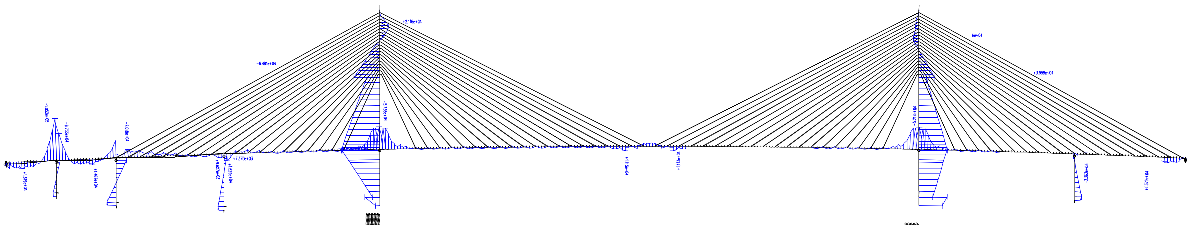


Fig.17 Construction stage 81 after applying permanent loads, bending moments in the superstructure & pylon: moments are as defined in the ADDCON Method

6. Conclusions

A method to find the optimal tensioning strategy for the construction of cable-stayed bridges has been derived. This paper explains this method called the Addcon Method and explains how non-linear and time-dependent effects which are relevant for the design of bridges can be included. The Addcon Method computes the correct tensioning forces for the stay cables which lead exactly to a pre-determined moment distribution within the deck and the pylon and also to the intended geometry of the bridge rendering the traditional trial-and-error approach to this problem obsolete. The method has been implemented into a bridge-design software package and has been used in practice on several occasions. One of these practical applications, the analysis of the Verige Bridge in Montenegro, serves as an example in this paper. The method is not restricted to bridge design, many other applications exist as the method has been formulated and implemented in a very general way.

7. References

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