Computer Based Optimising of the Tensioning of Cable-Stayed Bridges

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Summary

A numerical approach to reduce the calculation effort when attempting to minimise the number of stressing operations during the erection of cable-stayed bridges is shown. The proposed method is illustrated with sample calculations from a small example and from the Uddevalla bridge which is currently under construction.

The Problem

The solution for the optimum tensioning strategy for long span cable-stayed bridges can be an extremely tedious and time-consuming process for the following reasons:

Practical reasons

- Tensioning one cable affects the forces in all the other cables
- Cables can not, in reality, withstand compressive forces but stressing an adjacent cable may apparently cause this condition.
- Stressing of the stay cables is an expensive procedure due to the difficulty in the cable stressing procedure.

Analytical reasons

- A minimal cable tensioning strategy whilst saving a considerable amount of time and money during the construction phase greatly complicates the analytical phase of the design process.
- Definition of the tensioning strategy is interrelated with the chosen erection method and the simulation of the erection procedure using the structural model can be very complicated.
- The deck girder and pylon system must behave reasonably during all phases of construction. i.e. the deflections should be neither excessive nor incompatible with type of construction.
- Creep and Shrinkage (applicable where some or all the bridge elements are concrete or partially concrete members) greatly complicates the analytical process.

- “Uplift conditions” could exist at the temporary supports which further complicates the analysis. Whilst special “Non-tension” members could be used, this greatly increases the degree of indeterminacy and hence the speed of analysis.

All of the above practical and analytical reasons obviate the need for a consistent, standard, non trial and error type approach to the solution for these complex structures. It is possible to use a unit load system of analysis tied to the bridge construction method, relate this to an estimate of the max/min final live load moment envelope and through this, where possible, minimise the cable stressing operations.

The proposed method will always achieve a solution, which must then be checked for structural consistency by the user. The results can be structurally unacceptable as the solution is directly achieved from a set of simultaneous equations. The structural unacceptability may arise from such things as “compression in the cables” or “unacceptably high stresses” etc.

- Structurally acceptable results clearly demonstrate that the parameters chosen to define the structure and its construction are correct and also define the required tensioning and construction strategy
- Structurally unacceptable results will point the way for modification of the parameters to be used in the next analysis. (The modification would typically be to the “Ideal Bending Moment Diagram” – Refer below)

Choosing the System and manipulating the Moment Diagram.

The basic bridging system must be chosen and optimised before the stressing strategy design can be found. The system is chosen through a series of considerations such as required bridge functionality, availability and cost of materials, Clients requirements etc. The bridging system is considered, from this analytical viewpoint, as basic given information.
Integral with the bridging system choice is the concept that almost any moment diagram can be achieved in the deck and in the pylon by adjusting the following Degrees of Freedom:

- The tensioning forces in the stay cables and their stressing procedure
- The support movements (translation – longitudinal and vertical)
- Prefabrication shape of the deck girder and the pylon
- The erection procedure of the deck and the pylon

Finding the “Ideal Moment Diagram” for dead load.

Once the basic information has been defined in principle, the effects of the traffic / pedestrian loads and any additional loading – balustrade / guard-railing / surfacing / etc. on this “fundamentally defined” structure can be estimated.
The load capacities of the deck sections and the pylon sections can then be compared with the live plus additional dead load envelopes and the “ideal” dead load force diagrams may then be defined. The sign of the “ideal” dead load force diagrams may well be of opposite sign to the diagrams resulting from normal load directions. This demonstrates the distinct advantage of cable supported structures where the initial dead load moment diagram can be easily manipulated to suit the design needs.

Establishing the “Unit Force Equations” Principles

General

When the “ideal” dead load force diagrams have been defined then the system of unit forces can be mathematically equated to these “ideal” dead load force diagrams.
The process for defining the tensioning sequence and amount, the deck and pylon construction sequence as well as any required deck/pylon prefabrication effects then begins.
Process Principle:

- The unit loading system is first defined for the final stage structure in order to establish reasonable member sizes. This process usually involves certain re-definitions of the member sizes and program re-runs to prove structural integrity.
- Once reasonable values have been achieved the “unit force method” can be extended to the construction stage analysis.
- Each construction stage can be checked and proven for design compliance.

Degrees of Freedom

The most commonly selected unit forces or Degrees of Freedom (N.B these “DOF´s” are not the same as the structural system DOF´s) in the structural system include:

- A unit shortening of the cable (causing an axial cable tension) – or a unit tensioning causing an axial cable shortening.
- A unit translation of a rigid support (transverse or longitudinal movement at a pier or abutment support). – A longitudinal force applied at the end of the deck changes the moments by changing the cable forces which act on the deck.

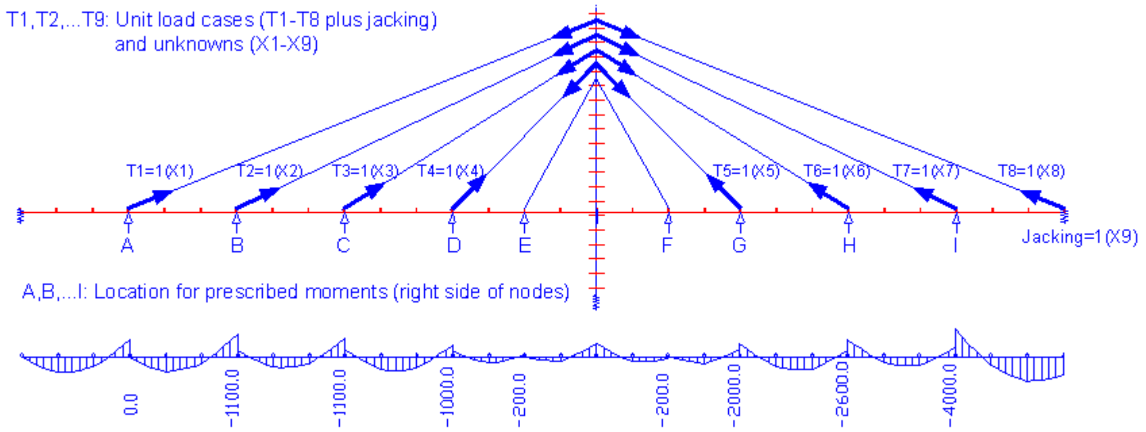
Setting up the “Unit Force Equations”

- Define the unit loading cases and the “ideal moment diagram”. The same number of unit loading cases must be defined as the number of “Fixed Moment” points chosen on the structural model to represent the “ideal moment diagram” (or vice-versa!).
  Principles in Example below:
- The “ideal dead load bending moment diagram” is defined for the deck girder by bending moments at 9 points along the girder (positions A, B, C ...... I).
- Nine unit loading cases are selected for setting up the simultaneous equations.
  - The 8 unknown “required” stay cable forces – chosen in this case to be 1000 kN.
  - One unit translation at the end support – chosen in this case to be 50 cm settlement.

The solution to the equations (the unknowns) will be the factor by which the unit loads should be factored to achieve the “ideal dead load bending moment diagram”.

Note: There is no fixed prescription for the selection of the unit loading cases. The designer is free to choose whatever he wishes. This flexibility is demonstrated in the example by the non-selection of the two stay cables adjacent to the pylon (positions E and F).
The 1000 kN unit cable force was selected to be of the same order of magnitude as the final cable force because of 2nd Order considerations. (Refer below for further description of 2nd Order effects.)
The “ideal moment diagram” for dead load is given below and is very different from the dead load bending moment diagram (M_P) which would result if the loading was applied to the structure with un-stressed cables.

N.B. Care must be taken in the selection of sensible and unrelated “ideal moments” as if one is related to another (i.e. dependent on it) then a singularity in the equations will result and there will be no solution. Provided there is no singularity, a solution will be reached.



The following system of linear equations is set up:

$M_A = M_P + M_{T1=1} \, . \, X_1. + M_{T\,2=1} \, . \, X_{2} + \ldots\ldots\ldots \; M_{T\,8=1} \, . \, X_{8} + M_{T\,J} \, . \, X_{9}$

. .

. .

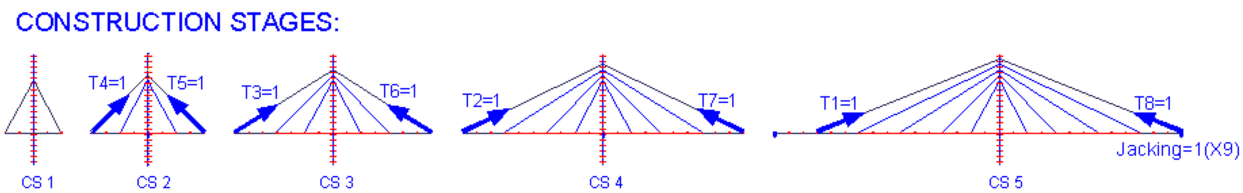
$M_I = M_P + M_{T1=1} \, . \, X_1. + M_{T\,2=1} \, . \, X_{2} + \ldots\ldots\ldots \; M_{T\,8=1} \, . \, X_{8} + M_{T\,J} \, . \, X_{9}$

$M_{A\ldots\ldots}$ M_I	Final stage moment at the current position (including tensioning + jacking).
M_{P}	Permanent load moment at the current position (without tensioning or jacking).
$M_{T1=1\ldots}$ $M_{T\,8=1}$	Bending Moment due to each unit tensioning at the current position.
M_J	Bending Moment due to unit jacking of the end support at the current position.

The X_{1} ...... X_{9} factors set up in the unit loading cases are the unknowns in the set of linear equations which are found by the solution of these equations.
Note that the system of equations is not symmetrical and the diagonal coefficient may be zero. This is to be considered when solving the equations. This basic solution defines the cable forces and the jacking force for the final stage and, at the moment, does not include the effects of: the sequence of construction stages, the creep, 2nd Order Theory or the non linearity of the cables due to the sagging effects.
The basic principles must therefore be extended to accommodate these effects.

Construction Stage analysis.

A similar system of unit loading cases can be defined for the construction stage analysis. The unit loading cases are, in this case, applied to the different structural systems which exist at the individual construction stages. The sketches below, which show a few of the construction stages, demonstrate the principle of the method of analysis. The loading cases for each construction stage are combined to form the set of simultaneous equations which must be solved to find the required multiplication factors for the unit loadings.



Creep – a rational linear approach

There is a general belief that creep design is a non-linear problem and therefore it is often approached in an empirical manner using some "rules of thumb" or "past experience" to assess its affects. This approach is particularly prevalent where the structural concrete is subjected to the many and varying loads which occur during the multiple construction stages of large cable-stayed bridges.
Taking account of creep effects using the CEB-FIP model code is even more complex than was the case using more traditional methods. Inspite of the above statements it can be shown through a series of mathematical equations that the effects of creep can be treated in a linear manner.
The derivation for the effects of creep is founded on the known fact:

$$\{\varepsilon_e\} \cdot \phi = \{\varepsilon_c\} \qquad \text{(Elastic Strain * Creep Factor = Creep Strain)}$$

Decomposing the structure down to element level, the above equation is applied to each individual element by applying the generalised displacement method rules for calculating initial strain type loads:
Define $\{\varepsilon_e\}$ over the element.
Define $\{\varepsilon_c\} = \{\varepsilon_e\} \cdot \phi$ over the element.
The member end displacements $\{\delta_c\}$ are found by weighted integration of the strain vector over the element length in the usual way.
The member end forces are calculated and the system of equations are assembled and solved for nodal displacement $\{\delta\}$ in the usual way.
$(\{\delta\} - \{\delta_c\}) \cdot [k] = \{F_I\}$ gives the internal forces due to creep.
The system of analysis is completely linear up to this point.
In the specific case of creep, cognisance must be taken of the age differences in the concrete as well as the various ages of different parts of the structure at the time of each increment of load application (ε_e is no longer constant but varies with time). A finite difference approach in time is applied here and using a linear variation over a time interval, we can say:

$$\{\varepsilon_t\} = \{\varepsilon_0\} \cdot \frac{t_1 - t}{\Delta t} + \{\varepsilon_1\} \cdot \frac{t - t_0}{\Delta t}$$

This equation can then be put into the basic displacement equation: at time t_0, $\{\varepsilon_e\}$ is known and then by solving the equations for $\{\delta\}$ at time $t = t_1$ a recursive formula can be derived which results in a linear relationship at time t_1 such that the equation including the effects of creep is the same as the original equation with the exception that the modulus E is replaced by

$$\frac{E}{1+\phi*0.5}$$

A detailed description of the whole procedure is given in Ref. 2 and the theoretical background for the finite difference approach to solve "initial value problems" is described in Ref. 3.
The essence of the above statement is, that all the creep influences on the final distribution of internal forces and displacements are related in a linear manner to the elastic strain which itself initially caused the creep.
The principles of linear superposition may, in consequence, be applied and the total creep occurring during a single time step may be decomposed into single contributions:
Considering one of the prescribed ideal moment positions:
$M_{creep} = M_p + M_{c\,t=1} \cdot X_1 + M_{c\,t=2} \cdot X_2 + M_{c\,t=3} \cdot X_3$ ....etc.
M_{creep} therefore consists of one part which is related to the permanent load and the other parts are related to the unit loads described above which are linearly coupled to the same unknown factors X_1 ...... X_9.
As before the basic concept can now be applied; the effects of creep for permanent loads and for unit loads are decomposed into separate contributions from each time interval and then summed. The system of equations for defining X_1 ...... X_9 therefore remains linear. The only

approximation made is the assumption that the behaviour within any single time step is linear which is consistent with the usual application of the "finite differences in time" approach.

Second Order Theory and cable non-linearity (due to cable sag)

Since the element stiffness depends on the axial force (in the case of 2nd Order Theory as well as for cable sag), the basic displacement method equations become non-linear. The equations defining the solutions for X_1 ...... X_9, which were proved to be linear for the creep case above, also become non-linear. An iterative approach must therefore be applied:

The Simple Approach

- Estimate X_1 ...... X_9, and use this estimate of the unknowns to find the variable stiffness which on substitution into the equations hopefully gives a solution which is close to the final behaviour.
- Correct the estimate of X_1 ...... X_9 and calculate again.

A Better Approach

Use the tangent stiffness for calculating the influence of the application of a small increment to each unit loading case. The equations can then be transformed to define the iterative correction for X_1 ...... X_9 and a procedure such as the Newton Raphson method can be set up.

The tangent matrix for the 2nd Order Theory or even large deflections (with respect to suspension bridges) can be similar to that usually applied in the "Large Deflection Theory". E.g the corrective term N/L is added into the appropriate position in the element stiffness matrix. The cable sagging effects can be accommodated by deriving $d_S/d_{\Delta x}$ from the well known "Peterson Formulae" (Ref. 4). Where S means the Cable force and Δx is the cable extension. Convergency is accelerated and guaranteed, when using the tangent matrix with the Newton Raphson approach as long as a real solution exists.

The Results from the sample analysis.

This particular example was chosen not only to demonstrate the principles of analysis but also to demonstrate the effects of 2nd Order Theory and of creep on the structure.

The results from a few selected points have been chosen for demonstrating these principles:

Final Stage cable forces (kN) resulting from the different analyses

Cables	1st order theory & creep	2nd order theory – no creep	2nd order theory & creep
Posn E	1073.9	1079.8	718.6
Posn F	1000.8	1003.5	663.9

Initial Stage cable forces (kN) resulting from the different analyses

Cables	1st Order Theory & creep	2nd Order Theory – no creep	2nd Order Theory & creep
Pos^n B	1775.52	1484.44	1833.65
Pos^n I	1788.02	1468.22	1840.82

Pylon Moments (kNm) resulting from the different stage analyses (design system)

Construction stage	1st Order Theory & creep	2nd Order Theory – no creep	2nd Order Theory & creep
1	-500.00	-41	-503.0
2	-471.0	-32	-472.0
3	-466.2	-33	-464.0
4	-463.1	1493	-538.0
5	-2151.6	-617	-2413.0
Final	-2019.0	-617	-1977.0

Pylon Moments (kNm) resulting from the different stage analyses (1st system)

Construction stage	1st Order Theory & creep	2nd Order Theory – no creep	2nd Order Theory & creep
1	0	-	0
2	0	-	0
3	0	-	0
4	10492.4	-	15657.0
5	10651.0	-	15567.9
Final	244.1	-	1126.0

Minimum Deck Girder Moment Envelope (kNm) (design system)

Construction stage	1st Order Theory & creep	2nd Order Theory – no creep	2nd Order Theory & creep
1	-4338	-4851	-4347
2	-4129	-4877	-4223
3	-4194	-4835	-4228
4	-3951	-5018	-3982
5	-3792	-5089	-3819

Maximum Deck Girder Moment Envelope (kNm) (design system)

Construction stage	1st Order Theory & creep	2nd Order Theory – no creep	2nd Order Theory & creep
1	1876	1238	1935
2	3232	2420	3401
3	2906	2756	3098
4	3675	2860	3775
5	3906	3146	4018

These above results highlight:

- The importance of accurate creep action assessment and shows that creep effects are critical to the structural integrity and must be accurately calculated and can not simply be assessed from some “arbitrary rules”. It can be seen that the creep in the deck affects the cable forces which in turn affect the deck and pylon moments significantly. The pylon moments are modified to such a degree that they are even reversed in construction stage 4.

- The importance of consistent construction stage checks as the moments in the pylon, whilst being quite acceptable in the final stage are excessive in the construction stage under the 1st system of analysis.
- The significant changes to the pylon moments caused by 2nd Order effects. (The pylon is highly compressed and therefore sensitive to additional moments from the deflected shape).
- The easy parameter design check:
  - Whilst a solution to the 1st system of analysis was found, the pylon failure in construction stage 4 & 5 was easily identified.
  - Inspection of the 1st system showed that the translational fixity at the pylon was the cause of the excessive moments. Removal of this fixity proved to be an adequate modification to the design system.

Construction Stage Analysis – forwards or backwards?

The traditional method of carrying out the construction stage analysis is to start at the “Final stage structure” and gradually reduce the structure (going backwards) stage by stage until the first construction stage is reached.
It is argued that this method is the most likely to achieve the fastest result as it starts from a structurally correct solution - the final stage – which may possibly have been defined using the unit load method described above. Whilst this argument does have much merit, it falls down when problems are subsequently found at a particular construction stage. The check then reduces to a trial and error method. Using the proposed unit load method, the forwards or backwards solution are equally possible and equally simple as even the creep principles described above can be adapted for the backwards solution by solving “the equations” for ε_0 instead ε_1.

The Uddevalla Cable-Stayed Bridge

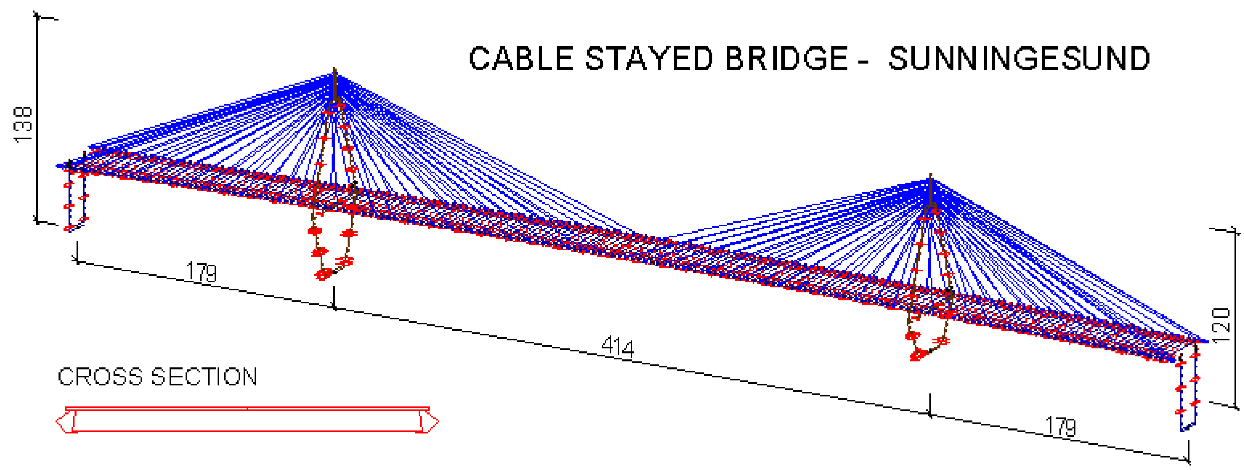
A bridge which was designed using the principles described above is the Uddevalla Cable-Stayed Bridge.
This cable-stayed bridge is the central part of a continuous 1712m crossing over the Sunninge-sund waterway between Udevallamotet north and Udevallamotet south in Sweden.
The approach viaducts, comprising twin steel box girders with a concrete slab, are rigidly connected to the main bridge on either side and provide overall longitudinal structural stability.
The cable-stayed bridge portion comprises a 414 metre main span, symmetrical side spans of 179 metres and two 85 metre high (above the deck girder) diamond shaped concrete pylons which anchor the fan shaped stay cable arrangement. The stay cables, which support the bridge deck on either side, are anchored at 13.32 metre centres in the longitudinal direction.
The bridge deck structure carries 6 lanes of traffic and comprises a composite, open steel grid structure with a 240 mm thick concrete top slab which spans longitudinally over the diaphragms. The deck edge beams (I type beams) also have a thin walled shell structure connected to the side which in addition to acting as a wind spoiler provides some torsional stiffness to the edge beams.
More comprehensive descriptions and details of the Uddevalla Cable-Stayed Bridge can be found in Ref. 4.
Given below is a summary of the principles used in the analysis of the Uddevalla Cable-Stayed Bridge using the unit load method:

The “Degrees of Freedom” (or unknowns) chosen for the unit load analysis were:

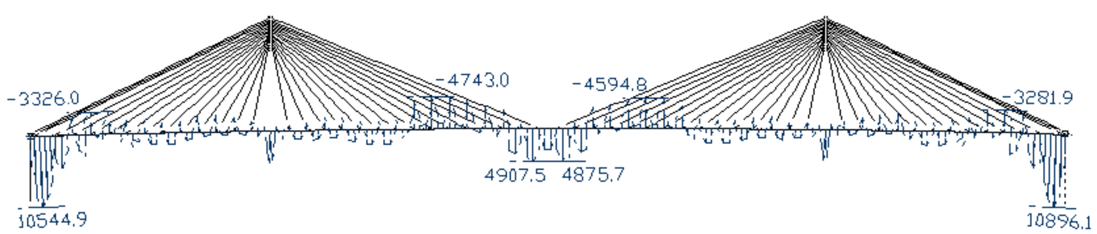
- All the stay cables – a unit tensioning
- Translation at one cable-stayed bridge pylon support (“X” and “Y” directions)

The Uddevalla Cable-stayed Bridge construction requires a 3 stage stressing procedure:
Stage I stressing provides support for the new steel portion of the deck during assembly. The cables are initially stressed to provide support and to counteract excessive deflection before making the welded connection to the existing deck.
Stage II stressing provides support for the whole structural self weight comprising the steel plus pre-cast concrete top slab elements. The procedure is simple as the stressing jacks from the previous stressing operation are still connected. The first Unit load analysis to find the cable forces is carried out at this stage.
Stage III stressing is required for counteracting the superimposed dead load and creep effects on the pylon deflection. The procedure is required because the stringent minimal pylon moment criteria precludes a sufficient pylon pre-camber. The second Unit load analysis to find the re-tensioning cable forces is carried out at this stage.

The “Ideal Moment diagram” chosen for the initial dead load is shown below together with a general bridge arrangement. Note the unusual shape in this “Ideal Moment diagram” in the deck girder was dictated by a strict limitation prescribed for the pylon moments which takes cognisance of the “very severe” environmental conditions for reinforced concrete weathering/corrosion. In order to comply with this stipulation, the “Ideal Moment diagram” had to include a minimal moment condition in the pylon.

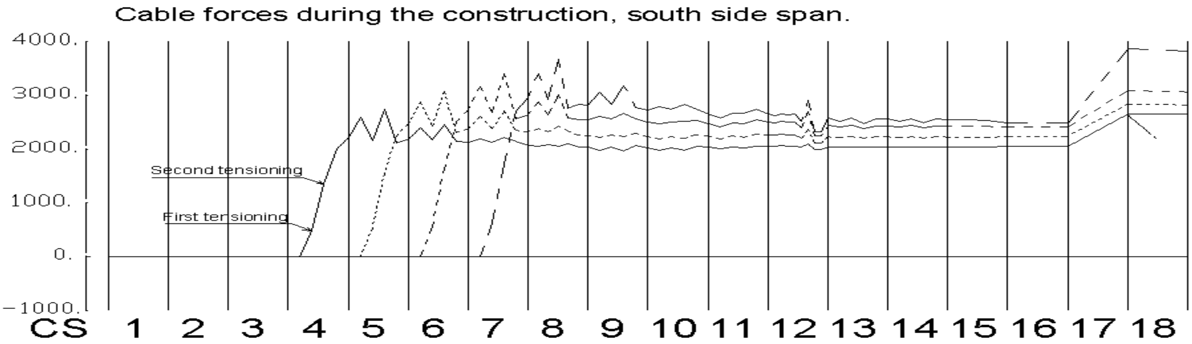


Ideal Moment Diagram

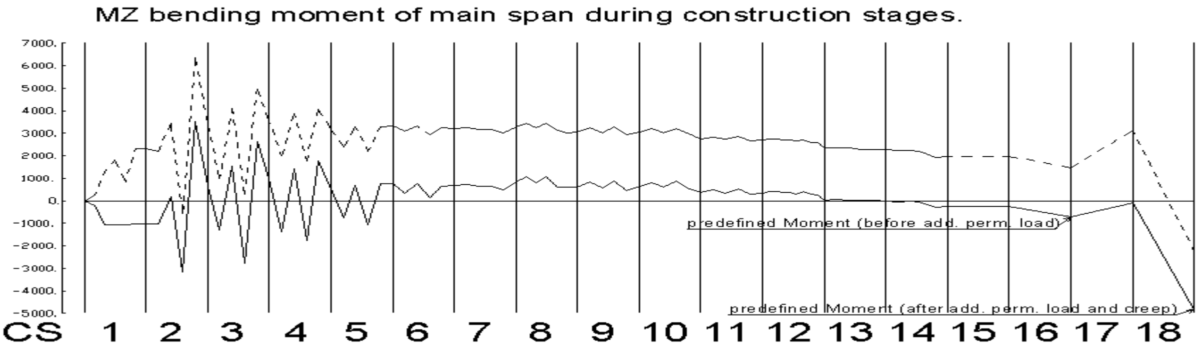


The diagrams below show the time dependency of a few characteristic results from the analysis. The time axis is not to scale but shows a sequence of the different actions. Stages 1-16 are the cable tensioning and deck cantilevering stages. The deck construction is complete at the end of stage 16, the additional dead load is applied in stage 17 and stage 18 is for creep and shrinkage up to "time infinity".

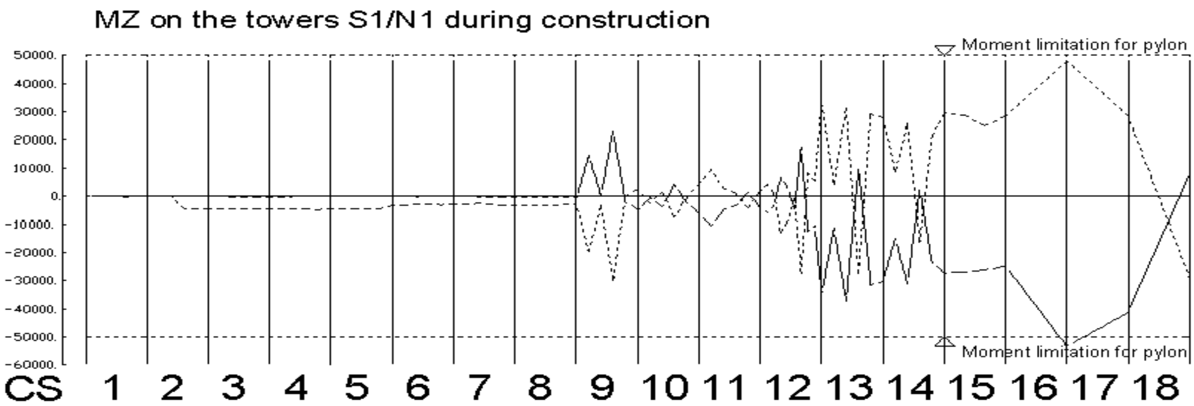
Cable force variation in side span cables 5, 6, 7 and 8 (numbering from the pylon)



Main span moment variation at cable 2 north and cable 2 south (numbering from the pylon)



Moment variation in pylon at the top of the footing



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